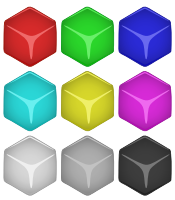
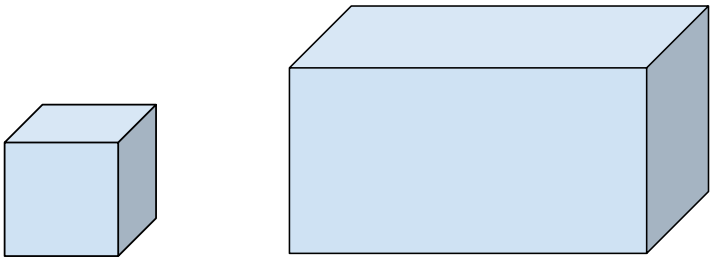


Problem of the Week
Problem A and Solution
Block Builder

Problem

Tabia has 18 blocks. Show the number of different solid, rectangular prisms can she build, using all 18 cubes.



Note: A rectangular prism is a 3-D figure with six rectangular faces.

Solution

Since the rectangular prism is formed by solid objects, each dimension of the prism must be a whole number of blocks. According to the question, the volume of the prism is 18. The volume is calculated by multiplying the three dimensions together. So you need to find the total number of distinct combinations of three whole numbers that have a product of 18.

To find these combinations, look at the numbers that divide exactly into 18. They are: 1, 2, 3, 6, 9, and 18. Pick any two of these numbers and determine if there is a third value from this list so that the product of the three numbers is 18. There are four combinations that will work:

- $1 \times 1 \times 18 = 18$
- $1 \times 2 \times 9 = 18$
- $1 \times 3 \times 6 = 18$
- $2 \times 3 \times 3 = 18$

Note that a rectangular prism with dimensions $1 \times 3 \times 6$ is the same as a rectangular prism with dimensions $3 \times 1 \times 6$. The objects are simply rotations of each other.

Teacher's Notes

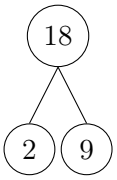
In this problem, you are given 18 blocks and there are four different rectangular prisms you can assemble with those blocks. Interestingly, a larger number of blocks does not necessarily lead to a larger number of prisms. For example, given 35 blocks, there are only two possibilities: $1 \times 1 \times 35 = 35$ and $1 \times 5 \times 7 = 35$.

The number of different prisms that can be formed depends on the prime factorization of the number of blocks. A prime factorization is a product formed entirely of prime numbers. A prime number has exactly two factors: 1 and itself, and 2 is the smallest prime number. For example, the prime factorization of 48 is: $2 \times 2 \times 2 \times 2 \times 3$.

One way to find the prime factorization of a number is to use a factor tree. Here is how you can form a factor tree for the number 18.

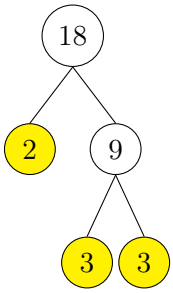
Start with the number 18.

Since 18 is not a prime number, find a factor pair, and add two *branches* that lead to *nodes* (the circles) containing each number of the factor pair.



Look for any other *nodes* in the tree that are not prime numbers. Add two branches from those nodes that lead to *nodes* containing each number of a factor pair. Continue adding new *branches* and *nodes* to the tree as long as there are non-prime number nodes. In this case, the only non-prime number left is 9 with a factor pair 3×3. At this point, the factor tree for 18 is complete.

The prime factorization is composed of the nodes in the tree that contain prime numbers. So, the prime factorization of 18 is: $2 \times 3 \times 3$.



More factors in the prime factorization of a number means more possible divisors for that number, which means there are more ways you can write the number as a product of three numbers. *Highly composite numbers* are a special set of numbers with many divisors. They were first described in detail by Srinivasa Ramanujan, a mathematician from India, born in 1887, who unfortunately died at the age of 32. Despite very limited opportunities for higher education early in his life, Ramanujan eventually went on to be a well-respected mathematician. Eventually he went on to continue his research at the University of Cambridge. His life has been dramatized in the film *The Man Who Knew Infinity (2015).*